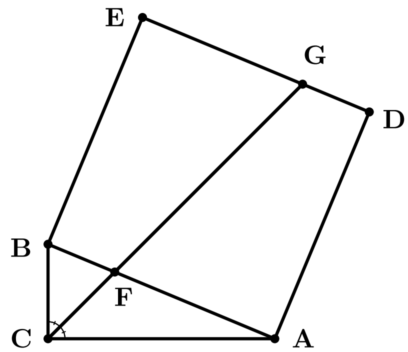
E-1/A: In triangle ABC the angle at C is a right angle. On it hypotenuse lies a square $ABCD$. The angle bisector at C intersects BA at F and ED at G. If $CA = 24$ and $CB = 10$, what is the area of the quadrilateral $ADGF$?

(15 points)



E-1/B: The hexagon $ABCDEF$ has all angles equal. We know that four consecutive sides of the hexagon have lengths 7, 6, 3 and 5 in this order. What is the sum of the lengths of the two remaining sides?

(15 points)

E-1/C: Find that H rectangle with unit area that has minimal perimeter and to which there exists an H_1 rectangle the perimeter of which is smaller than the perimeter of H by 50 % and the area of which is bigger than the area of H by 50 %.

The answer you need to give is the forth power of the sum of the length of the sides of H.

(20 points)

E-1/D: A + B + C 70 points

E-2/A: If we divide the positive integer k by a prime p the remainder is 6. We also get 6 as a remainder if we divide $1000 - k$ by p. We know that $10000 - k$ is divisible by p. What is the value of p?

(10 points)

E-2/B: In a french village the number of inhabitants is a perfect square. If 100 more people moved in, then the number of people would be 1 bigger than a perfect square. If again 100 more people moved in, then the number of people would be a perfect square again. How many people live in the village if their number is the least possible?

(15 points)

E-2/C: Poppy the elephant was playing with her friend Toby. Poppy wrote the following numbers in the snow: $1, 1/2, 1/3, \ldots 1/10$. Toby can delete two numbers from the snow, a and b, but then he needs to write $a + b + ab$. Ha repeats this operation 9 times after which there's only one number left. What's the largest possible value of the remaining number?

(20 points)

E-2/D: A + B + C 70 points

E-3/A: 99 irrational numbers are in a set. From the set we choose n numbers such that the sum of any two is irrational. What is the highest value of n that works for any set of 99 irrational numbers?

(10 points)

E-3/B: Samson writes the number 123456789 on a piece of paper . Then between any two adjacent digits he can add a multiplication symbol, even several to different places, or none altogether. The digits between two multiplication symbols are interpreted as one integer, so he will get a product of some integers, for example $1234 \times 56 \times 789$. What is the last four digit of the largest possible value he can receive?

(20 points)

E-3/C: Solve the equation $3^x + 4^x + 5^x = 6^x$ on the set of real numbers.

The answer you need to give is the sum of the root(s).

(20 points)

E-3/D: A + B + C 70 points

E-4/A: Alice wants to build a labyrinth in Wonderland. She designed it on checkered paper: She drew a big grid square (its vertices are grid points and the sides are parallel to the grid lines). Then, inside the square she drew the lines symbolizing the walls, such that the sum of the lengths of walls became 400 units.(she always connected grid points with lines parallel to the grid lines.) When she was done, she realised that there is exactly one route between any two unit squares. (A valid route contains each unit square at most once.) How long is the side of the big square she drew first (in units)?

(10 points)

E-4/B: A cube has been divided into 27 equal-sized sub-cubes. We take a line that passes through the interiors of as many sub-cubes as possible. How many does it pass through?

(15 points)

E-4/C: $P(x) = x^4 - (k+3)x^3 - (k-11)x^2 + (k+3)x + (k-12) = 0$. We know that two roots of $P(x)$ are independent of k. What is the smallest possible positive k such that the other two roots are real?

(25 points)

E-4/D: A + B + C 70 points

E-5/A: At a wedding dinner, some of the members of a table of six know each other. The best man asks the members how many people from said table do they know. The five numbers spoken by the first respondent are all different. What is the maximum number of people the sixth person at the table can know?

(10 points)

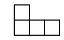
E-5/B: The hazelnut-raisin chocolate consists of 100 cubes, each cube containing a whole number of hazelnuts and raisins. What is the smallest number k such that you can eat k cubes of any of these bars of chocolate and eat at least half of the hazelnuts and raisins?

(15 points)

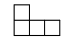
E-5/C: How many 10-digit numbers exist such that all of its digits are different, and the difference between any two consecutive digits is at most 2?

(25 points)

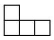
E-5/D: A + B + C 70 points

M-1/A: In how many different ways can we cover a 4×6 table with the following 4-square big L-shape tiles?  (We can rotate/reflect the tiles. We consider two coverings different if there exists 4 squares that are covered with 1 tile in one covering but not in the other.)

(15 points)

M-1/B: In how many different ways can we cover a 4×8 table with the following 4-square big L-shape tiles?  (We can rotate/reflect the tiles. We consider two coverings different if there exists 4 squares that are covered with 1 tile in one covering but not in the other.)

(20 points)

M-1/C: In how many different ways can we cover a 4×10 table with the following 4-square big L-shape tiles?  (We can rotate/reflect the tiles. We consider two coverings different if there exists 4 squares that are covered with 1 tile in one covering but not in the other.)

(25 points)

M-1/D: A + B + C 80 points

M-2/A: We place one plane onto each edge of a unit cube and each of these planes fulfills the following:

1. It does not intersect the interior of the cube.

2. The angle between the plane and either of the two faces of the cube fitting onto the same edge of the cube is 45 degree.

What is the volume of convex solid figure defines by the 12 planes placed as described above?

The answer you need to give is the square of the volume.

(15 points)

M-2/B: Point P, circle k, and AB secant containing P are given, such that PA=PB=1. The tangents from P to k are touching the circle in point C and D. The intersection of AB and CD is M. What is the length of PM?

The answer you need to give is the floor of $10 \times PM^2$.

(25 points)

M-2/C: Roo and Kanga drew a pretty triangle in the sand. Roo measured one of its altitudes, it was 9 cm. Kanga measured another altitude to be 29 cm. The Antelope of Altitudes measured the third altitude, that was m centimeters (m is an integer). What is the product of the smallest and the largest possible value of m?

(30 points)

M-2/D: A + B + C 100 points

M-3/A: A hiker decided that she will walk m kilometers in n ($n \geq 2$) days. (m is a positive integer.) On the first day, she walked 1 kilometer and one seventh of the remaining distance, on the second day she walked 2 kilometers and one seventh of the remaining distance, and so on. In the end, on the last day (the nth day) she walked the remaining distance which was exactly n kilometers. How long was the hike (in kms)?

(20 points)

M-3/B: Inside the squares of a 2023×9100 table we wrote the positive integers from 1 to $2023 \cdot 9100$ in increasing order. We did that two times. The first time we were filling in the rows from left to right, starting with the upmost row than going down. The second time we filled in the columns from up to down, starting with the leftmost column then going right. How many squares did we get there that have the same number written inside them twice?

(25 points)

M-3/C: Seven classmates are comparing their end-of-year grades in 12 subjects. They observe that for any two of them, there is some subject out of the 12 where the two students got different grades. It is possible to choose n subjects out of the 12 such that if the seven students only compare their grades in these n subjects, it will still be true that for any two, there is some subject out of the n where they got different grades. What is the smallest value of n for which such a selection is surely possible?

Note: In Hungarian high schools, students receive an integer grade from 1 *to* 5 *in each subject at the end of the year.*

(30 points)

M-3/D: $\mathbf{A + B + C}$ 100 points

M-5/A: How many ways are there to arrange the numbers 1, 2, 3, ..., 15 in some order such that for any two numbers which are 2 or 3 positions apart, the one on the left is greater?

(20 points)

M-5/B: There are red and blue balls in an urn: 1024 in total. In one round, we do the following: we draw the balls from the urn two by two. After all balls have been drawn, we put a new ball back into the urn for each pair of drawn balls: the colour of the new ball depends on that of the drawn pair. For two red balls drawn, we put back a red ball. For two blue balls, we put back a blue ball. For a red and a blue ball, we put back a black ball. For a red and a black ball, we put back a red ball. For a blue and a black ball, we put back a blue ball. Finally, for two black balls we put back a black ball.

Then the next round begins. After 10 rounds, a single ball remains in the urn, which is red. What is the maximal number of blue balls that might have been in the urn at the very beginning?

(25 points)

M-5/C: Santa Claus plays a guessing game with Marvin before giving him his present. He hides the present behind one of 100 doors, numbered from 1 to 100. Marvin can point at a door, and then Santa Claus will reply with one of the following words:

- ”hot” if the present lies behind the guessed door,
- ”warm” if the guess is not exact but the number of the guessed door differs from that of the present's door by at most 5,
- ”cold” if the numbers of the two doors differ by more than 5.

At least how many such guesses does Marvin need, so that he can be certain about where his present is?

Marvin does not necessarily need to make a ”hot” guess, just to know the correct door with 100% certainty.

(30 points)

M-5/D: $\mathbf{A + B + C}$ 100 points

H-1/A: There are two numbers: F and G. We know that $F_a = 0,3737\ldots = 0,\dot{3}\dot{7}$ and $G_a = 0,7373\ldots = 0,\dot{7}\dot{3}$, where F_a and G_a are the forms of the numbers F and G in a Base-a numeral system. We also know that $F_b = 0,2525\ldots = 0,\dot{2}\dot{5}$ and $G_b = 0,5252\ldots = \dot{5}\dot{2}$, where F_b and G_b are the forms of the numbers F and G in a Base-b numeral system. Determine the value of $a+b$ (in base 10).

(30 points)

H-1/B: The pages of a book of 60 pages are numbered 1, 2, ..., 120. However, a few pages from the book was lost a few days ago. The sum of the numbers written on remaining pages is 7159. How many pages have been lost?

(40 points)

H-1/C: Let $\mathcal{S}$ be the set of all positive integers less than 10,000 whose last four digits in base 2 are the same as its last four digits in base 5. What remainder do we get if we divide the sum of all elements of $\mathcal{S}$ by 10000?

(45 points)

H-1/D: A + B + C 130 points

H-2/A: We place  L-shape triminos on a 8×8 board, they cannot overlap. What is the smallest number of triminos we can put on the board such that it is impossible to put down any more?

(20 points)

H-2/B: On each square of a 4×4 board we put a white or a black knight such that all knights can see at least one knight of both colours. (Meaning that it is 1 knight-move away from both a white knight and a black knight in.) How many different arrangements exist that satisfy the aforementioned conditions?

(35 points)

H-2/C: The numbers from 00 to 99 is written on a 10×10 board as shown on the figure. A knight starts its tour on 00, and finishes it on 99, but it can only step on cells in which the number is divisible by 3. It can't step on a previously visited cell. How many cells are visited during the longest such tour? (Including 00 and 99.)

(40 points)

90	91	92	93	94	95	96	97	98	99
80	81	82	83	84	85	86	87	88	89
70	71	72	73	74	75	76	77	78	79
60	61	62	63	64	65	66	67	68	69
50	51	52	53	54	55	56	57	58	59
40	41	42	43	44	45	46	47	48	49
30	31	32	33	34	35	36	37	38	39
20	21	22	23	24	25	26	27	28	29
10	11	12	13	14	15	16	17	18	19
00	01	02	03	04	05	06	07	08	09

H-2/D: A + B + C 120 points

M-4/A: Let's call a positive integer pretty if it can be written as a sum of some (at least two) consecutive positive integers. What is the sum of the first 11 positive integers that are NOT pretty?

(20 points)

M-4/B: A famous man – born in the last century – in 1999 was just as old as the sum of the squares of the digits of his year of birth. When was he born? (Note: if somebody was born in, say, 1929, he would be 70 years old in 1999).

(25 points)

M-4/C: Find the smallest positive integer n with the following property: if we write down all positive integers from 1 to 10^n and add together the reciprocals of every non-zero digit written down, we obtain an integer.

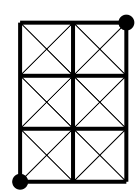
(30 points)

M-4/D: A + B + C 100 points

H-3/A:

In Giza the 6 highest ranking officials received a pyramid. The 6 pyramids were built next to each other in a 2×3 formation as you can see in the diagram below. The pyramids have square bases, their faces are equilateral triangles with a side-length of 18 m. A bug (that can walk on the faces of the pyramids) is in the top-right corner and wants to reach the opposite corner in the shortest route. (the two corners are marked on the diagram) If the length of the shortest route is d meters, what is d^2?

(35 points)



H-3/B: The sides of a non-rectangular triangle are denoted by the usual notation a, b, c, and the angles opposite them are α, β, γ. What will be γ (in degrees) if $\beta = 2 \cdot \alpha$, and the following equation holds up for the sides of the triangle:

$$\frac{a}{b} + \frac{b}{c} + \frac{c}{a} = \frac{a}{c} + \frac{b}{a} + \frac{c}{b}$$

.

(45 points)

H-3/C: The radii of the excircles of a triangle are r_a, r_b és r_c, the radius of its circumcircle is R. We know that $r_a + r_b = 3R$ and $r_b + r_c = 2R$. What is the smallest angle of the trianlge?

(50 points)

H-3/D: A + B + C 120 points

H-4/A: Roo is playing with his little cubes; he wants to build a bigger cube using all of those. He has 25 white and only 2 red little cubes, and he decided that the red cubes cannot touch each other, not even with their edges or vertices. How different great cubes can he get? (Two great cubes are different if he cannot get them from each other with only using rotations.)

(30 points)

H-4/B: There are 12 chairs arranged in a circle, numbered from 1 to 12. How many ways are there to select some of the chairs in such a way that our selection includes 3 consecutive chairs somewhere?

(40 points)

H-4/C: We have a deck of 49 cards, the cards are coloured 7 different colours, in each colour we have 7 cards numbered from 1 to 7. Bella randomly pulls 8 cards from the deck. We know that among the 8 cards she has pulled every colour and every number occurs at least once. What is the probability of Bella being able to discard a card such that among the remaining 7 cards all colours and all numbers occur? Give your answer in $100P + Q$ form, where $\frac{P}{Q}$ is the simplest form of the faction!
E.g. if the probability is $\frac{3}{10}$, *you should answer* $100P + Q = 310$.

(40 points)

H-4/D: $\mathbf{A + B + C}$ 150 points

H-5/A: 2 out of 10 metal balls are radioactive. One measurement of any number of balls can only determine whether or not there are any radioactive balls amongst them. (If there are, one measurement cannot determine whether there are one or more of them.) At least how many measurements must be made to determine which of the 2 balls out of 10 is radioactive?

(35 points)

H-5/B: Consider a number line consisting of all positive integers greater than 7. Marvin the storm-trooper traverses the number line, starting from 7 and working his way up. He checks each positive integer n, and paints it red if and only if (n choose 7) is divisible by 12. As Marvin travels through the number line the fraction of checked numbers that are painted red approaches a limiting number p. If p can be written in the simplest form r/q, what is the value of q?

(40 points)

H-5/C: 2022 people sit around a round table. We give Andrew a candy. Then we give a candy to the person sitting on the right of Andrew. Then we give a candy to the one who sits (1+2) seats away from Andrew (on the right), then to the one who sits $(1 + 2 + 3)$ seat away and so on... in the end we give a candy the person who sits $(1 + 2 + \cdots + 2022)$ to the right from Andrew. How many people have received candies?

(50 points)

H-5/D: $\mathbf{A + B + C}$ 150 points