Problems connected the lecture about poles and polars

1. **(Géza's remark)** Let S be a sphere, choose a plane containing the center of the sphere, and choose line ℓ in the plane such that it does not intersect the sphere. Let us call the intersection of the plane and the sphere circle γ. We call points P and Q on line ℓ conjugates if P is on the polar of Q with respect to circle γ. Let T be a point on the sphere such that the plane of points P, Q and T touches sphere S.
   Prove that angle $\angle PTQ$ is a right angle.

2. **(Connected to the second Romanian Master's problem)** Quadrilateral $ABCD$ is incribed circle γ. Let the intersection of lines AB and CD be point P, of lines BC and DA be point Q. Let M be the midpoint of segment PQ.
   Prove that the length of the tangent segment from M to circle γ is the same as the length of segment PM (and QM).

3. Points A, B, C, D, E and F are on circle γ in this order. The tangents at A and D, and lines BF and CE are concurrent. Prove that lines AD, BC and EF are concurrent or parallel to each other.

4. Convex quadrilateral $ABCD$ has an inscribed circle called γ. Lines AB and CD meet at point P, lines BC and DA meet at point Q. We connect the opposite points where γ touches the sides of $ABCD$: the two resulting segments meet at R. The center of γ is O. Prove that OR is perpendicular to PQ.

5. Let UV be a diameter of a semicircle, P and Q two points on the semicircle such that P is closer to U than Q. Let the intersection of PU and QV be point S, the intersection the tangents and P and Q be R.
   Prove that RS is perpendicular to UV.